07-04-2009

Applications of Quantum Physics

Motivate your answers
Answers may be given in Dutch

μ_B= 9.3 x 10^{-24} [J/T], k = 1.38 x 10^{-23} [J/K], 1 [J] = 1.6 x 10^{-19} [eV]

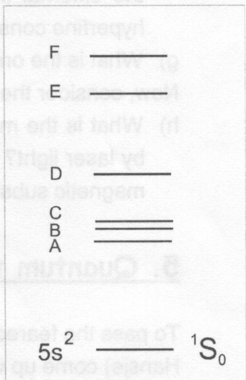
1. Binding Energies

Consider a calcium atom Ca($1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$).
For reference, the binding energy of H($1s$) is 13.6 eV.

a) Calculate the ionization potential (in eV) of this atom. The quantum defect is δ=2.51.
b) Calculate the effective charge experienced by the 4s electron.
c) The total binding energy of the two 4s electrons together is 18.0 eV. Calculate the quantum defect describing the binding energy of the 4s electron in Ca^+($1s^2 2s^2 2p^6 3s^2 3p^6 4s$).
Why is the quantum defect calculated at b) larger, equal or smaller than the one given at a)

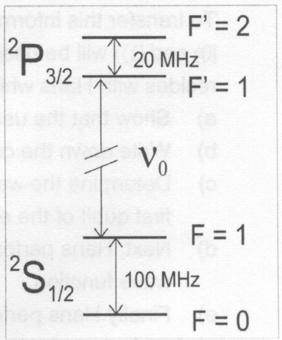
2. Configurations, Terms, and States

The figure on the right shows schematically the binding energies of the 7 lowest levels of an atom with a (.....$5s^2$) $^1S_0$ ground state.



a) Assume LS coupling to determine the nomenclature of the 6 excited levels (A-F).
b) How would the figure change if JJ coupling would apply.

3. Doppler-free saturation spectroscopy



a) Briefly describe and/or sketch the method of Doppler-free saturation spectroscopy.
b) Sketch the Doppler-free saturation spectra of atoms of which the electronic structure is depicted in the figure.
c) Calculate to which velocity classes of atoms the peaks correspond.

4. Magnetic Resonance Imaging using ^{129}Xe

For MRI diagnostics of interstitial cavities hyperpolarized atoms as ^{3}He and ^{129}Xe are used increasingly. Here ^{129}Xe will be considered. ^{129}Xe has a nuclear spin of $I = 1/2$ and a nuclear magnetic moment of $\mu = -0.78\ \mu_n$.

a) What is the value of g_I.

b) Sketch the behavior of the Xe(.....$5p^6$) ground state as a function of magnetic field and indicate the relevant quantum numbers.

c) For a magnetic field of $B = 0.5$ T, calculate the energy splitting between the states.

d) At this field of 0.5 T, calculate the difference in population (in %) between the relevant states at room temperature (T=300 K)

e) Why is it of importance for MRI diagnostics to increase the difference in population.

To increase the population difference between nuclear spin states one lets the Xe atoms interact with spin-polarized Rb. In these interactions spin alignment is transferred from the Rb to the Xe. The following deals with the nuclear spin polarization of Rb by optical pumping between the Rb(5s $^{2}S_{1/2}$) groundstate and the excited Rb($5p$ $^{2}P_{3/2}$) state.

For Rb I=3/2 and g_I=1.8, g_J(5s $^{2}S_{1/2}$) = 2 and g_J($5p$ $^{2}P_{3/2}$) = 4/3.

f) Sketch the behaviour of the Rb(5s $^{2}S_{1/2}$) states as a function of magnetic field and indicate the relevant quantum numbers. Consider B = 0, B = "weak" and B = "strong". Recall that in the "strong"-field case the formula for the level shifts contains besides terms depending on the external field, a term which is external-field independent, namely Am_Jm_I (with A the hyperfine constant).

g) What is the origin of this term Am_Jm_I.

Now, consider the laser pumping between Rb(5s $^{2}S_{1/2}$) and Rb($5p$ $^{2}P_{3/2}$) at B=0.

h) What is the most suitable hyperfine transition to be used to obtain nuclear spin orientation by laser light? Does the laser need to be polarized. Make a schematic drawing showing the magnetic substates and indicate the induced and/or spontaneous transitions.

5. Quantum teleportation

To pass the feared "applications of quantum physics" exam two fictitious students (Hans and Hansje) come up with the devious plan to exchange information via quantum teleportation. One way or the other Hans knows the answer to problem 5, namely that the "answer" qubit (with eigenstates $|0\rangle$ and $|1\rangle$) has the wavefunction $\alpha|0\rangle - \beta|1\rangle$ ($\alpha^2+\beta^2=1$).

To transfer this information to Hansje an entangled state of two particles (each with eigenstates $|0\rangle$ and $|1\rangle$) will be used. The entangled state they use is $2^{-½}\{|01\rangle + |10\rangle\}$ of which the first qubit resides with Hans while the second one is send to Hansje.

a) Show that the used state $2^{-½}\{|01\rangle + |10\rangle\}$ is indeed entangled

b) Write down the combined wave function of answer and entangled state

c) Determine the wave function after Hans has performed a controlled-NOT operation on the first qubit of the entangled state with the "answer" qubit as control bit.

d) Next, Hans performs a Hademard gate operation on the "answer" qubit. Determine the wave function.

e) Finally Hans performs a measurement and finds $|00\rangle$. Which operation has Hansje to perform on her wavefunction to get the correct answer $\alpha|0\rangle - \beta|1\rangle$.